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LETTER TO THE EDITOR

Translational and orientational order of the flux-line lattice of a high-T_c superconductor

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Abstract. We show that non-local elastic effects in high-T_c superconductors strongly modify the range of translational and orientational order in a pinned flux state. Decoration experiments at low fields and temperatures should therefore be interpreted using non-local elasticity theory. Fitting correlation lengths determined this way to the data gives an estimate of the pinning strength. At higher fields the local theory is valid; with this estimate of the pinning strength the correlation length is at least 10^3 lattice spacings, suggesting that a lattice phase will be observed at low temperatures, where our theory is valid.

The discovery of high-T_c superconducting materials has stimulated renewed interest in the properties of flux-line lattices (FLL) [1]. These materials have relatively short superconducting correlation lengths $\xi \sim 10$ Å, while magnetic penetration depths λ are a hundred times larger. Thus, the Ginzburg–Landau parameter $\kappa = \lambda/\xi$ is very large. This feature has been shown by several authors [2–4] to make considerations of *non-local* elasticity [5] important, considerably softening the FLL. The effect of such intrinsic softening may have been seen in decoration experiments [6]. In particular, there are predictions that the melting temperature of the FLL can be suppressed well below the mean-field $H_{c2}(T)$ phase boundary [2–4]. The large mass anisotropy in these layered materials also plays an important role in the suppression of the melting temperature [2].

In [2] and [4] the effects of pinning of the flux lines (FLs) by random impurities were not considered, but as shown by Larkin and Ovchinnikov (LO) [7], the random forces due to arbitrarily weak pinning destroys the long-range translational order of the FLL. Short-range order persists up to some correlation lengths $\xi_\perp$ and $\xi_\parallel$, along directions perpendicular and parallel to the applied field, respectively. (We consider only the geometry where $\boldsymbol{B} \parallel \hat{c}$.) On length scales greater than $\xi_\parallel$ and $\xi_\perp$ the lattice description breaks down, and it has been argued [8] that a vortex glass phase, with long-range orientational order [9] exists. Recently, magnetic decoration experiments [10] have been performed on high-quality single crystals of $Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_8$ (BSCCO) at low fields and temperatures. These experiments find translational correlation lengths of a

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few lattice constants and orientational correlations of at least several tens of lattice constants.

LO [7] determined $\xi_\perp$ and $\xi_\parallel$ from non-local elasticity theory for an isotropic 3D material at $T=0$. (A mistake in the calculation was subsequently corrected by Wördenweber and Kes, and also by Brandt [11].) In this paper we extend the results of [7] to include both thermal fluctuations and mass anisotropy. We show that correlation lengths arising from *non-local* elastic terms should be used to discuss the data from the decoration experiments rather than correlation lengths determined from *local* theory [12]. This is a surprising result, considering that the vortex separation in decoration experiments is typically $\sim\lambda_{ab}$, the in-plane magnetic penetration depth. The origin of this result lies in the extreme mass anisotropy of the materials.

The amplitude of the correlation function is reduced by thermal diffuse scattering. This effect is negligible at the temperatures and fields at which the decoration experiments are carried out. Fitting the non-local correlation lengths to the decoration data gives a value for the strength of the pinning, which allows us to predict correlation lengths up to fields $H_0 \sim 4(M/M_z)\Phi_0/d^2$, the limit of validity of 3D elasticity theory. Here M_z and M are quasiparticle masses in the ab-plane and along the $\hat{c}$-axis respectively, $\Phi_0 = 2.07\times10^{-7}\,\mathrm{Gcm}^2$ is the quantum of flux and d is the interplanar spacing; for BSCCO $H_0 \approx 1$ T. At fields $\geqslant 0.25$ T, much larger than those applied in the decoration experiments, we find that the local correlation lengths are smaller than the corresponding non-local ones. Thus, in this field regime, local elasticity theory is applicable. However, fitting the decoration data to the non-local theory yields local correlation lengths which are substantially larger than lengths obtained by fitting the data to local theory. We then show that the asymptotic value of the orientational correlation function is strongly modified by non-local elasticity.

Before we present results, we emphasize that at fields ~ 60 G or larger, topological defects are practically non-existent in the FLL in well-annealed samples [10, 13]. Under such circumstances, we expect a treatment in which elastic restoring forces are assumed to dominate plastic deformation forces to be well suited. For lower fields it is clear that topological defects of the FLL proliferate, and an inclusion of such defects (e.g. dislocations, disclinations, dislocation loops) into the description is necessary. This is outside the scope of the present letter, which will analyse the simpler situation presented by the field region $B \geqslant 60$ G. Finally, a note of caution: all comparisons made between theory and experiment here assume that the decoration experiments are carried out on FLLs that are in thermal equilibrium, and not some frozen-in non-equilibrium configuration.

The details are as follows. We determine *translational* correlation lengths from the correlation function $g(r) \equiv [\langle \exp(\mathrm{i}G\cdot(u(r)-u(0)))\rangle]_{\mathrm{Av}}$. Here, G is a reciprocal lattice vector of the triangular FLL, and the 2D displacement field, u, is a measure of the distortion of the FLL from its Abrikosov ground state. The angular brackets denote a thermal average over the Hamiltonian

$$H = \tfrac{1}{2}\sum_k u_i(-k)\Phi_{ij}(k)u_j(k) - \sum_k u(k)\cdot f(-k)$$

$$\times\ \Phi_{ij}(k) = c_{\mathrm{L}}(k)k_ik_j + \delta_{ij}[c_{66}k_\perp^2 + c_{44}(k)k_z^2]. \qquad (1)$$

Here, $\Phi_{ij}(k)$ is the elastic matrix, c_{66}, $c_{\mathrm{L}}(k)$ and $c_{44}(k)$ are the shear-, bulk-, and tilt-moduli, respectively [2], $k_\perp^2 = k_x^2 + k_y^2$, and $(i,j)\in(x,y)$. Note that the dispersion of $c_{\mathrm{L}}(k)$ and $c_{44}(k)$ is important, even at very low inductions in anisotropic superconductors

[23], (see (3)). The square brackets $[\ ]_{Av}$ indicate a quenched statistical average over the random pinning force $f(k)$, which is assumed to have a Gaussian probability distribution. At zero temperature the variance of the pinning force, $W = n_p\langle f_p^2\rangle$, can be taken to have the form $W = \bar{W}b(1-b)^2$. Here $b = B/B_{c2}$, and B_{c2} is the upper critical field. At any finite temperature $\bar{W}$ will be field and temperature dependent [14–16]. A Gaussian probability distribution for the pinning force allows an exact evaluation of $g(r)$, which can be written as a product, $g(r) = g_T(r)\, g_W(r)$. Here, $g_T(r)$ is the thermal correlation function in the absence of pinning, and $g_W(r)$ is the correlation function due to pinning at $T = 0$. At large distances, g_T is given by the Debye–Waller factor $\exp(-G^2\langle u^2\rangle/2)$, where $\langle u^2\rangle$ is given by (3.2) of [2]. The function $g_W(r)$ is given by

$$g_W(r) = \exp\left(-W\sum_k (1-\cos k\cdot r)G_i\, A_{ij}(k)G_j\right)$$

$$A_{ij}(k) = P_{ij}^T/[c_{66}k_\perp^2 + c_{44}(k)k_z^2]^2. \qquad (2)$$

In (2), $P_{ij}^T = \delta_{ij} - k_ik_j/k_\perp^2$, and we have ignored a term involving *longitudinal* fluctuations of the FLL, since $c_L(0) \gg c_{66}$, and the dominant contributions to the k-sum in (2) come from small k. The exponent in (2) is essentially the LO-correlation function [7] $\langle(u(r) - u(0))^2\rangle$. With the non-local elastic moduli for the FLL in a uniaxial material derived in [2], $g_W(r)$ can be evaluated to give†

$$g_W(r) = \exp\{[-G^2W/16\pi c_{66}^{3/2}c_{44}^{1/2}(0)](f^{\rm l}(r) - f^{\rm nl}(r))\}$$

$$f^{\rm l}(r) = [r_\perp^2 + c_{66}z^2/c_{44}(0)]^{1/2}$$

$$f^{\rm nl}(r) = (\Gamma/4k_h)\ln[1 + k_{\rm BZ}^4r_\perp^4 + (c_{66}/c_{44}(0))(k_{\rm BZ}^4/k_h^2)\Gamma^2z^2]. \qquad (3)$$

In (3), we have defined $\Gamma = (M_z/M)^{1/2}$. Note the large prefactor of the logarithm in (3), making non-local elasticity important *even at very low inductions*. This has its origin in the magnetic field components parallel to the *ab*-planes, with very long range, which originate in the tilted vortices [3]. In (3), $c_{66} \doteq (B_{c2}^2/4\pi)b(1-b)^2/8\kappa^2$ and $c_{44}(0) = B^2/4\pi$ are *local* elastic moduli and $k_{\rm BZ}$ is the radius of the circularized Brillouin zone of the FLL. We have $k_{\rm BZ} = (2b)^{1/2}/\xi$, $k_h = (1-b)^{1/2}/\lambda_{ab}$, and $G = (2\pi/(3)^{1/2})^{1/2}k_{\rm BZ}$ [1–4]. As $g_W(r)$ decays to zero, the decay of the full correlation function $g(r)$ will be determined by $g_W(r)$ provided that the Debye–Waller factor is sufficiently large, say $\sim e^{-1/2}$, i.e. $\langle u^2(r)\rangle = 0.04a^2$. This is the case [2] at the temperatures $T = 4$ K and fields $H \approx 100$ G at which the decoration experiments [10] are carried out.

Local elasticity is recovered when $k_h \to \infty$; $g_W(r)$ will decay exponentially with correlation lengths $\xi_\perp^{\rm local} = 16\pi c_{66}^{3/2}c_{44}^{1/2}(0)/G^2W$, and $\xi_\parallel^{\rm local} = 16\pi c_{66}c_{44}(0)/G^2W$. The *non-local* terms in the elastic moduli of the FLL lead to the logarithm in (3); the importance of this term was pointed out in [11]. We now examine its role in BSCCO, where $\kappa \approx 100$ and $\Gamma^2 \approx 3600$. Comparing $f^{\rm l}$ and $f^{\rm nl}$ in (3), we find that in the transverse direction the *non-local* term dominates if $k_{\rm BZ}r_\perp < (2b/(1-b))^{1/2}\Gamma\kappa\ln k_{\rm BZ}r_\perp$. Taking $B_{c2} \approx 500$ kG, and $\Gamma \approx 60$ [17] for BSCCO, we find that at fields ~ 100 G, this is the case if $k_{\rm BZ}r_\perp < 500$, or $r_\perp < 100\,\mu$m. This distance is much larger than measured correlation lengths [10]

† The LO correlation function $\langle(u(r) - u(0))^2\rangle$ for an *isotropic* superconductor was generalized slightly in [11] by including a Labusch parameter in the elastic matrix $\Phi_{ij}(k)$. This parameter describes the elastic interaction of a homogeneously shifted FLL with the pins. In our equilibrium statistical mechanical treatment, which we believe adequately describes the experimental systems we claim to consider, no such parameter can be generated, since the system *retains translational invariance* after one has averaged over the random disorder.

(and is arrived at with a lattice spacing of 1.4 μm at 20 G [18], decreasing with field as $\sim 1/b^{1/2}$).

Defining correlation lengths arising from the non-local terms in $g_W(r)$ by retaining the logarithm only, and equating the resulting exponent to -1, we obtain

$$\xi_\perp^{\text{non-local}}/a = 0.26 \exp(1.9 \times 10^{11}\, b^{1/2}(1-b)^{3/2} B_{c2}^4 \xi / \Gamma \kappa^4 \bar{W})$$
$$\xi_\parallel^{\text{non-local}}/a = [1.6/\Gamma(1-b)^{1/2}](\xi_\perp^{\text{non-local}}/a)^2. \qquad (4)$$

As long as the non-local correlation lengths are smaller than their local counterparts, the range of $g(r)$ will be determined by (4) although the actual functional form will be a power law rather than an exponential. Fitting (4) at 100 G to three lattice spacings [10], we determine the strength of the pinning directly from the decoration experiments. We find $\bar{W} = 2.84 \times 10^{-3}\,\text{N}^2\,\text{m}^{-3}$. Thus, for weak fields $b \ll 1$, $\xi_\perp^{\text{non-local}}/a = 0.26 \exp(1.72 \times 10^2 b^{1/2})$. This yields correlation lengths of 1.45, 3 and 5.11 lattice spacings at fields of 50 G, 100 G and 150 G, respectively. By comparison, at this value of $\bar{W}$, the correlation length deduced from the local theory, $\xi^{\text{local}}/a = 0.38 \times 10^6 b$, is much larger, ~ 100, in this field range. Having determined $\bar{W}$, we can also predict the power law associated with the decay of $g(r) \approx r^{-\alpha}$. We find $\alpha = 0.58 \times 10^{-2}\, b^{-1/2}$. In this field range the aspect ratio of a translationally ordered domain $\xi_\parallel/\xi_\perp = 1.6/[\Gamma(1-b)^{1/2}]$ $(\xi_\perp/a)$, grows exponentially with $b^{1/2}$.

At fields larger than 100 G, the non-local correlation length initially grows exponentially with $b^{1/2}$. However, as a result of this rapid initial growth, there will be a wide range of fields for which $\xi_\perp^{\text{local}} < \xi_\perp^{\text{non-local}}$, and at these fields the *local* theory will determine the range of $g(r)$. From (4), we find that this is the case for $B \geqslant 0.25$ T. In this field range the perpendicular correlation length $\xi_\perp^{\text{local}}/a = 0.71 \times 10^{11} b(1-b)$ $B_{c2}^4 \xi/\kappa^3 \bar{W}$, which is 10^3 lattice spacings, 2×10^2 μm, at 0.25 T, grows slowly with magnetic field, suggesting that a lattice phase will be observed over a wide range of fields. Had we fitted the low-field decoration data to the local correlation length, the resulting value of $\xi_\perp^{\text{local}}$ would have been *two orders of magnitude smaller*. These conclusions hold at fields $\leqslant H_0$ and at temperatures less than a few degrees.

At lower temperatures, $\leqslant 4$ K, the effect of g_T becomes important at fields of the order of 10 kG. At higher fields than this we extract correlation lengths by equating $g_W(r)$ to e^{-1} divided by the Debye–Waller factor. When the Debye–Waller factor is approximately one-half, the local correlation length will be reduced by at least an order of magnitude, whereas the correlation length deduced from the non-local theory would be only reduced by a factor of two. Thus, our zero-temperature estimate of the field range in which local theory is applicable should remain essentially unchanged. We expect, then, a perpendicular correlation length of about 10^3 lattice spacings, corresponding to about 100 μm at fields of order 10 kG. At this field strength the parallel correlation length, including the effect of g_T will be of order 0.01 cm, which by far exceeds the typical sample thickness.

The *orientational* order of the FLL was considered recently by Chudnovsky [12]. Using *local* elasticity theory to study the orientational correlation function $g_6(r) \equiv [\langle \exp[6i(\Theta(r) - \Theta(0))]\rangle]_{\text{Av}}$ at large r, he showed that (in the case discussed here, of a pinning force coupled directly to the FLL displacement) the lattice retains long-range orientational order. A random field coupled directly to the bond angle $\Theta(r)$ will of course destroy the orientational order at sufficiently large distances [12, 19]. This general conclusion is not affected by the non-local elastic effects, but the detailed form of the orientational correlation function is. At long distances the correlation function at $T = 0$ takes the form

$$g_6(r) = \exp\{-[1.3 \times 10^{-11}\kappa^4\bar{W}/b^{1/2}(1-b)^{3/2}B_{c2}^4\xi][1-((1-b)\xi/\Gamma\kappa b r_\perp)]\}. \quad (5)$$

Given the value of $\bar{W}$ found here, at $B = 100$ G, $g_6(r)$ decays very rapidly over a distance of the order of $10^{-2}a$, to e^{-1}. Here again we see the importance of non-local elasticity, which introduces factors of κ and Γ into the exponent of the asymptotic value of $g_6(r)$ and, in addition, modifies the field dependence. The net effect is to enhance the exponent by a factor of $\sim 10^2$ relative to the local theory. By contrast, non-local effects do not modify the rate of decay of the correlation function to its asymptotic value. For $B = 60$–100 G, the magnitude and b-dependence of $g_6(r)$ agree well with experiments on well-annealed samples.

In summary, we have considered in detail the translational and orientational correlations of flux lines in a weakly pinned FLL using a *non-local* elastic description of the FLL. Our prediction of a power-law decay of translational correlations is in consistent agreement with low field (60–100 G) decoration experiments on well-annealed samples (although topological defects should be included at even lower fields). Whether or not the decoration data in this field regime really reveal exponential or power-law decay for the translational correlations remains an open question. More experimental work on a wider field range is needed to settle this [13]. Our predictions (4) for the correlation lengths at higher fields, outside the field regime where decoration experiments are feasible, can be tested by small-angle neutron scattering (SANS). A promising compound for this purpose seems to be $NbSe_2$, which has large values of κ and M_z/M, and for which large enough high-quality crystals can be obtained to provide good signal intensities.

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